REVIEW

Stress

- Stress on a surface is an internal distributed force system.
- The relationship of external forces (and moments) to internal forces and the relationship of internal forces and moments to the stress distribution are two distinct ideas.

Stress at a point:

---- Is an internal quantity.

---- Needs two directions and a magnitude to specify it.(2nd order tensor).

$$\sigma_{ij} = \lim_{\Delta A_i \to 0} \left(\frac{\Delta F_j}{\Delta A_i} \right)$$

---- Stress is a symmetric tensor.

---- Has units of force per unit area.

---- Sign is determined by the direction of the internal force and the direction of the outward normal of the imaginary cut surface.

Prefixes on stress: Normal stress, shear stress, principal stress, maximum shear stress, maximum in-plane shear stress, axial stress, bearing stress, bending stresses, torsional shearing stress, maximum normal stress, yield stress, ultimate stress, allowable stress, failure stress, etc.

Strain

- Measure of relative movement of two points on the body. (deformation).
- Elongations are positive normal strains.
- Decrease from right angle results in positive shear strains.
- Small Strain (< 0.01)

---- results in linear deformation analysis.

---- normal small strain are calculated using deformation component in the original direction of the line element, regardless of the orientation of the deformed line element.

---- the following approximations hold: $\tan\gamma \cong \gamma$ $\sin\gamma \cong \gamma$ $\cos\gamma \cong 1$ for small shear strain γ.

Strain at a point:

---- Is related to the first partial derivative of deformation.

---- a strain component is affected by the derivative direction and the deformation direction.

---- shear strain is symmetric

---- Tensor normal strain = Engineering normal strain; Tensor shear strain = Engineering shear strain/2;

Mechanical Properties

- Tension test is one of the most important test for determining material properties.

Material adjectives: linear material, elastic material, plastic material, ductile material, brittle material, hard material, tough material, strong material, isotropic material, homogenous material, etc.

Material constants: Modulus of elasticity (E), shear modulus (G), poisson's ratio (ν), tangent modulus, secant modulus, modulus of resilience, modulus of toughness, etc.

Generalized Hooke's Law for Isotropic materials:

- Relates stresses and strains in any orthogonal coordinate system.

$$\varepsilon_{xx} = \frac{\sigma_{xx}}{E} - \frac{\nu}{E}(\sigma_{yy} + \sigma_{zz}) \qquad \gamma_{xy} = \frac{\tau_{xy}}{G}$$

$$\varepsilon_{yy} = \frac{\sigma_{yy}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{zz}) \qquad \gamma_{yz} = \frac{\tau_{yz}}{G} \qquad G = \frac{E}{2(1+\nu)}$$

$$\varepsilon_{zz} = \frac{\sigma_{zz}}{E} - \frac{\nu}{E}(\sigma_{xx} + \sigma_{yy}) \qquad \gamma_{zx} = \frac{\tau_{zx}}{G}$$

- Plane Stress: Stresses with subscript z are zero.
- Plane Strain: Strains with subscript z are zero.
- The state of stress and the strain affects the third principal stress and strain and are important in the calculation of maximum shear stress and strain.

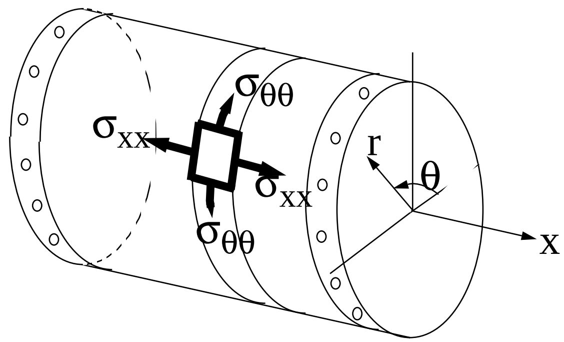
Factor of Safety

$$K = \frac{\text{Failure producing value of -----}}{\text{Estimated value of -------------}} \quad \text{[Load, Stress, Displacement]}$$

- Sudden changes in geometry, loading or material properties causes stress concentration.
- The effect of these sudden changes dies out rapidly as one moves away from the region of sudden changes (Saint Venant's Principle)

Thin Pressure Vessels

Cylindrical pressure vessel.



$$\sigma_{\theta\theta} = \frac{pR}{t} \qquad \sigma_{xx} = \frac{pR}{2t}$$

Spherical pressure vessels

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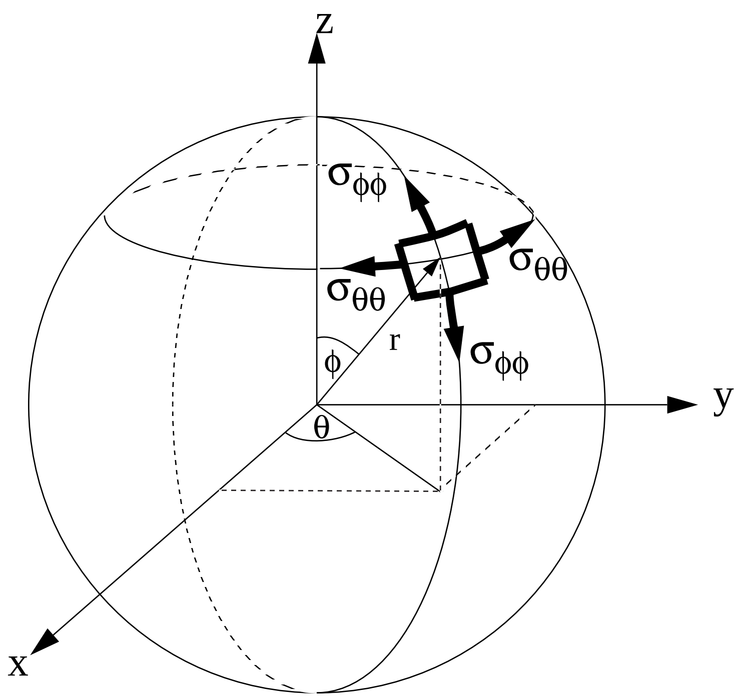
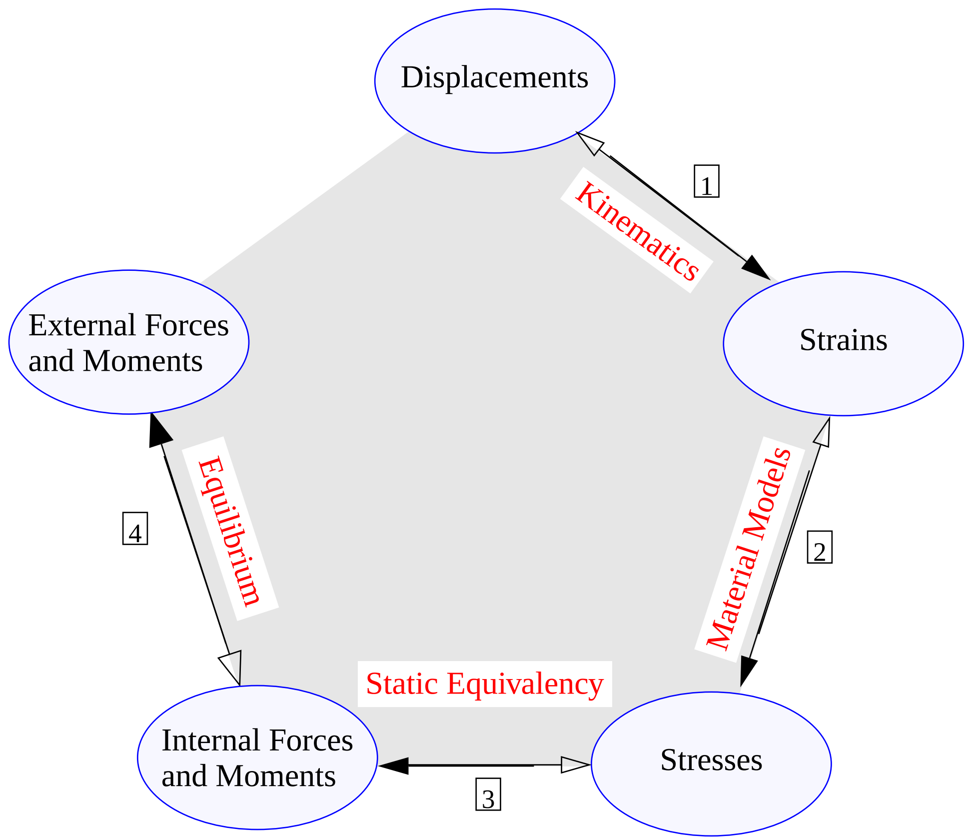
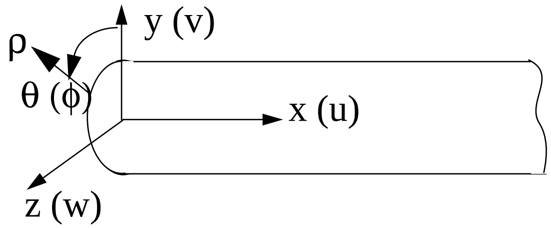


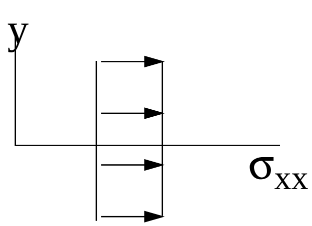
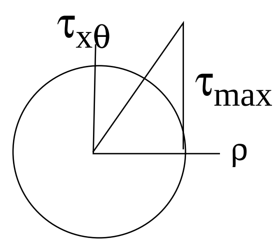
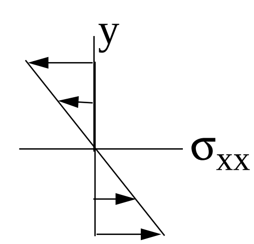
$$\sigma_{\theta\theta} = \sigma_{\phi\phi} = \sigma = \frac{pR}{2t}$$

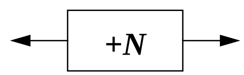
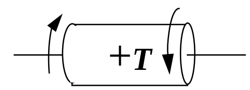
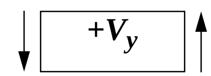
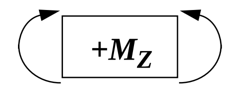
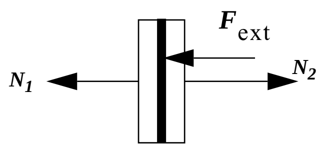
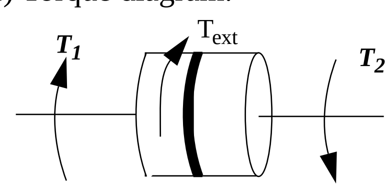
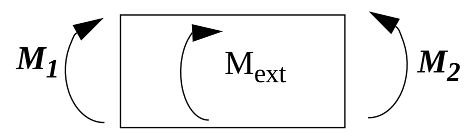
Structural Analysis



One-Dimensional Structural Theories and Formulas



	Axial (Rods)	Torsion (Shafts)	Symmetric Bending about z-axis (Beams)
Displacements	$u(x, y, z) = u(x)$ $v = 0 \quad w = 0$	$\phi(x, y, z) = \phi(x)$	$u(x, y, z) = -y\frac{dv}{dx}$ $v = v(x) \quad w = 0$
Strains	$\varepsilon_{xx} = \frac{du}{dx}$	$\gamma_{x\theta} = \rho\frac{d\phi}{dx}$	$\varepsilon_{xx} = -y\frac{d^2v}{dx^2}$
Stresses	$\sigma_{xx} = E\varepsilon_{xx} = E\frac{du}{dx}$ 	$\tau_{x\theta} = G\gamma_{x\theta} = \rho\frac{d\phi}{dx}$ 	$\sigma_{xx} = E\varepsilon_{xx} = -Ey\frac{d^2v}{dx^2}$ $\tau_{xy} \neq 0 \ll \sigma_{xx}$ 

	Axial (Rods)	Torsion (Shafts)	Symmetric Bending about z-axis (Beams)
Internal Forces & Moments	$\boldsymbol{N} = \int_A \sigma_{xx} dA$	$\boldsymbol{T} = \int_A \rho \tau_{x\theta} dA$	$\boldsymbol{N} = \int_A \sigma_{xx} dA = 0 \Rightarrow \int_A y dA = 0$ $\boldsymbol{M_z} = -\int_A y \sigma_{xx} dA \qquad \boldsymbol{V_y} = \int_A \tau_{xy} dA$
Sign Convention			 
Computation of Internal Forces & Moments	(1) Free body diagram. (2) Axial force diagram.  $\boldsymbol{N_2} = \boldsymbol{N_1} + F_{ext}$	(1) Free body diagram. (2) Torque diagram.  $\boldsymbol{T_2} = \boldsymbol{T_1} + T_{ext}$	(1) Free body diagram. (2) Shear-moment diagram.  $\boldsymbol{M_2} = \boldsymbol{M_1} + M_{ext}$
Stress Formulas	$\sigma_{xx} = \dfrac{\boldsymbol{N}}{A}$	$\tau_{x\theta} = \dfrac{\boldsymbol{T}\rho}{J}$	$\sigma_{xx} = -\left(\dfrac{\boldsymbol{M_z} y}{I_{zz}}\right) \qquad \tau_{xs} = -\left(\dfrac{\boldsymbol{V_y} Q_z}{I_{zz} t}\right)$
Deformation Formulas	$\dfrac{du}{dx} = \dfrac{\boldsymbol{N}}{EA}$ $u_2 - u_1 = \dfrac{\boldsymbol{N}(x_2 - x_1)}{EA}$ EA = Axial Rigidity	$\dfrac{d\phi}{dx} = \dfrac{\boldsymbol{T}}{GJ}$ $\phi_2 - \phi_1 = \dfrac{\boldsymbol{T}(x_2 - x_1)}{GJ}$ GJ = Torsional Rigidity	$\dfrac{d^2 v}{dx^2} = \dfrac{\boldsymbol{M_z}}{EI_{zz}}$ $v = \int\left[\int \dfrac{\boldsymbol{M_z}}{EI} dx\right] dx + C_1 x + C_2$ EI_{zz} = Bending Rigidity

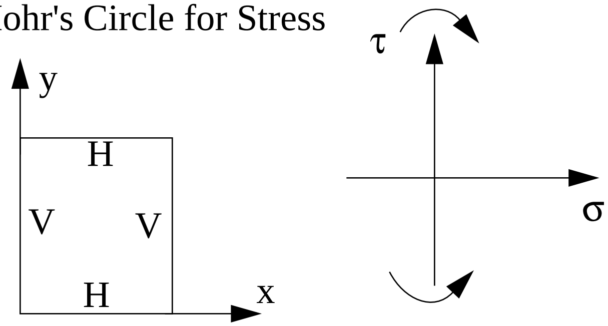
Stress transformation

- Relating stresses on different planes that pass through a point.
- Relating stresses in different coordinate system at a point.

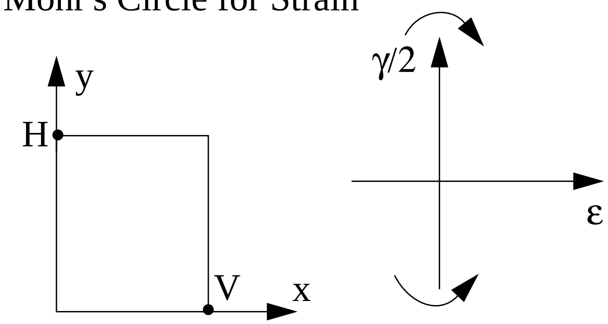
Stresses on various planes passing through a point in two dimension can be found by the:

---- Wedge method. Convert stresses to forces and use equilibrium equations to determine the unknowns.
---- Method of equations.
---- Mohr's circle for stress. A point on the circle represents a unique plane.

Mohr's Circle for Stress



Mohr's Circle for Strain



Strain transformation

- Relating strains in different coordinate system at a point.

Strains in different coordinate systems can be found by the:

---- Line method. Convert strains to deformation and use geometry and small strain approximation to determine the unknowns.
---- Method of equations.
---- Mohr's circle for strains. A point on the circle represents a unique direction.

Strain gages

- Strain gages measure only normal strains.
- Strain gages are stuck on free surfaces (plane stress).
- Strain gages measure average normal strains.
- A change in strain gage orientation by 180^{o} makes no difference in strain values.

Combined loading

- Stress components can be added (subtracted) to only other stress component in the same direction on the same surface at the same point.
- Identify the equations you need and use the equations as a check list of variables you need to calculate.
- Be methodical.

Design and analysis of simple structures

- Maximum (allowable) normal stress refers to principal stress and maximum (allowable) shear stress refers to the absolute maximum shear stress.
- Maximum bending normal stress is order of magnitude greater than maximum bending shear stress.
- Two major steps:
  - ---- (1) Analysis of forces and moment that act on individual members.
  - ---- (2) computation of stresses on members under combined loading.
- Plan before start solving and be methodical.

Buckling:

- Bending due to *compressive* axial forces is called buckling.
- It is sudden and usually catastrophic.
- Buckling occurs about the axis of *minimum* area moment of inertia.
- Euler Buckling Load P_{cr} can be calculated from: $P_{cr} = \frac{\pi^2 EI}{L^2}$
- Slenderness ratio is defined as L/r where L is length of column and r is radius of gyration.
- Critical slenderness ratio is where material failure and buckling failure can occur simultaneously.